OPTIMIZATION VEHICLE ROUTING PROBLEM WITH CROSS ENTROPY METHOD AND BRANCH & BOUND ALGORITHM

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ABSTRACT.

An alternate formulation of the classical vehicle routing problem (VRP) is considered for distributing fresh potatoes from warehouse to the food outlet or shop. We propose a new-heuristic (meta-heuristic) method to solve the problem, based on the Cross-Entropy method. In order to better estimate the objective function at each point in the domain, we incorporate Monte Carlo sampling. This creates many practical issues, especially the decision as to when to draw new samples and how many samples to use. We also develop a framework for obtaining exact solutions and tight lower bounds for the problem under various conditions by grouping the destination. This is used to assess the performance of the algorithm. Numerical results are presented for various problem instances to illustrate the ideas. Finally, solution obtained by Cross-Entropy compared with Branch and Bound Algorithm. It is proved that two methods have similar solution, with number of iteration was similar relatively.

Keywords: vehicle routing problem, cross-entropy method, branch and bound method.

1. INTRODUCTION

The vehicle routing problem (VRP) is an important problem that has been widely studied by logisticians, operations researchers and mathematicians for the last several decades. The problem was finding the set of routes that minimize two objectives simultanously: (1) the total number of vehicles required to serve all customer’s, and (2) total travelling cost; while satisfying the following three main constraints: (1) each route originates at the depot and terminates at one of the customers, (2) each customer is visited once and only once by exactly one vehicle and its demand is totally satisfied, and (3) the customers who are visited in each route have total demand less than or equal to the capacity of the vehicle assigned to serve the route. Several method’s can used to solve VRP are Branch and Bound (BB), Dynamic Programming, Hungarian or Konig’s Method, and newly method is Cross Entropy (CE).

The cross entropy (CE) method was conceived by Rubinstein (1997) as a way of adaptively estimating probabilities of rare events in complex stochastic networks. The method was soon adapted to tackle combinatorial optimization problems (Rubinstein, 1999 and 2001). Recently, the *Annals of Operations Research* devoted a volume to the cross entropy method (de Boer, et al. 2005a). Applications of the CE method to combinatorial optimization include vehicle routing (Chepuri and Homem-de-Mello 2005), the traveling salesman problem (de Boer, et al. 2005). As stated by de Boer, et al. (2005), the CE method, in its most basic form, is a fairly straightforward procedure that consists of iterating the following two steps: (1) generate a random sample from a pre-specified probability distribution function; and (2) use the sample to modify the parameters of the probability distribution in order to produce a “better” sample in the next iteration.

The Branch and Bound (BB) algorithm is the most widely used method for solving combinatorial optimization problem. Two models of combinatorial optimization problem widely used to solved by BB algorithm are TSP and machine scheduling problem (MSP). (Wiston, 1993). Hillier (2005) was formulate 3 step the BB algoritm, include for TSP problem : (1) branching, select the one that was created most recently; (2) bounding, obtain its bound by applying the simplex method; dan (3). Fathomed, for each new problem.

This research will try to solved TSP problem by CE method and BB algorithm. This paper will proofing whether or not the CE method, as represented the metaheuristic approach, and the BB algorithm, as represented the analytical approach, obtaining the similar solution. Process dan result of the two method/approach then was compared and evaluated. TSP problem on the paper was solved consist of 2 group selection by clustering, which each group consist of 6 nodes include with the warehouse. Because not practically to solved the problem by manual, in this research we using software Matlab version 2010 (CE method) dan Win QSB (BB algorithm).

2. PROBLEM TSP DESCRIPTION

A certain type of product is distributed from a factory to *N* customers, using two vehicles with two group’s tracks, having a fixed capacity *Q*. The vehicle strives to visit all the customers periodically to supply the product and replenish their inventories. On a given periodical trip through the network, on visiting a customer, an amount equal to the demand of that customer is downloaded from the vehicle; a reasonable assumption is that all the customers’ demands belong to a certain distribution (say normal) with varying parameters for different customers.

The vehicle sets out with the fixed capacity *Q*, and does not have knowledge of the demands that it will encounter on a given route, save for their individual probability distributions. Hence, there is a positive probability that the vehicle runs out of the product along the route, in which case the remaining demands of that customer and the remaining customers further along the route are not satisfied (a failure route). Such failures are discouraged with penalties, which are functions of the recourse actions taken. Each customer in the set can have a unique penalty cost for not satisfying the demand.

The cost function for a particular route travelled by the vehicle during a period is calculated as the sum of all the arcs visited and the penalties (if any) imposed. If the vehicle satisfies all the demands on that route, the cost of that route will simply be the sum of the arcs visited including the arc from the plant to the first customer visited, and the arc from the last customer visited back to the plant. Alternatively, if the vehicle fails to meet the demands of a particular customer, the vehicle heads back to the plant at that point, terminating the remaining route. The cost function is then the sum of all the arcs visited (including the arc from the customer where the failure occurred back to the plant) and the penalty for that customer. In addition, the penalties for the remaining customers who were not visited will also be imposed. Thus, a given route can have a range of cost function values associated with it. The objective is to find the route for which the *expected value* of the cost function is minimum compared to all other routes.

3. CE METHOD

The basic idea of CE method is to connect the underlying optimization problem to a problem of estimating *rare-event probabilities*, and use tools derived for that class of problems. The method has been shown to work quite well in the context of deterministic optimization; see, e.g., the tutorial paper by de Boer et al. (2005) and references therein. Note that we use here the CE method in the context of discrete *stochastic* optimization, so Monte Carlo techniques are needed to estimate the objective function. This creates many practical issues, especially the decision as to *when* to draw new samples and *how many* samples to use.

The CE method used in combination with the *importance sampling* (IS) technique. In those problems, the goal is to estimate the probability of occurrence of a very rare event. The difficulty lies in the fact that a standard Monte Carlo method will yield zero as the estimate, unless an extremely large sample size is used. Roughly speaking, the IS technique aims to select a probability measure that makes the occurrence of rare events more frequent, thereby reducing the variance of the estimator. It is known that an optimal zero-variance measure exists, but it is of impractical use since it depends on the quantities one wants to estimate.

In the CE method, the Kullback-Leibler cross-entropy is used to measure the distance between the optimal zero-variance measure and the importance sampling distribution. Thus, one chooses the distribution that minimizes that distance. The appeal of the method is that such minimization problem can be solved even though the optimal zero-variance measure is not completely known. This idea can be combined with and *adaptive* scheme, yielding a probability measure that can be used as an approximation of the optimal IS distribution. In Homem-de-Mello and Rubinstein (2005) the method is described in detail and some theoretical properties are established.

In Rubinstein (1999), the idea behind the CE technique is applied to combinatorial optimization. The key concept is to view the selection of an optimal solution at random from the domain of possible values as a *rare event*. More specifically, suppose we want to minimize a deterministic function $h(x)$ over a finite set X, and assume that h has a unique minimize x^*. Let $p(\cdot)$ be a uniform distribution on X, and let Y be a random variable on X with distribution p. Then, $\{Y = x^*\}$ is a rare event under p. As it turns out, the corresponding optimal IS distribution for this rare event is the atomic measure pointed at x^*. Thus, the CE method can be used as a heuristics to obtain the optimal solution. The resulting method has been applied successfully to numerous problems; we refer the reader to the tutorial by de Boer et al. (2005) for a more detailed discussion and references.

For demands following normal distributions, a variable-sampling scheme seems to work well. The optimal strategy seems to be to start at an appropriate size and then increase as $n \log(n)$ or $O(n)$, until stability is achieved and further increase of the sampling is not required. This scheme was implemented by starting with an initial sampling size of twice the problem size and increasing by steps of 10 linearly for a fixed number of iterations. This is because accuracy is not crucial at the early stages of algorithm. After this customary period, the sampling size is checked such that the half-width of a 95% confidence interval is not more than 1% of the sampling mean. If this is not the case, the sampling is increased; otherwise it is kept a constant. During the initial iterations, arriving at the exact value of the expected cost of a path is not so important. Thus, the sampling size can start at a smaller level and then be progressively increased. This increases the total number of iterations but the overall number of computations decreases.

Alternatively, we implemented a pre-processing routine to find a "good" sampling size a priori. Here, a path is randomly picked and the expected value of the path function is estimated using Monte-Carlo sampling. The sample size is taken to be very large (around 10,000) and the sampling size is brought down in steps such that the interval in the subsequent iteration always contains the interval of the previous iteration and the half-width of a 95% confidence interval never exceeds 5% percent of the estimated cost function value. Having picked the optimal sampling size using the above mentioned pre-processing routine, we start the CE procedure. At every iteration, we generate a sample with the given sample size. If the minimum cost function value's half-width exceeds it by 5%, we increase the sample size for the next iteration. Thus, the sampling size is still adaptive. This partially nullifies the possibility that the preprocessing scheme picked an unrepresentative path for sample size selection. The main advantage for the adaptive sampling size scheme is that we need not make a priori assumptions about the sampling size, which is a function of the structural complexity of the problem and hence, difficult to estimate.

4. TSP PROBLEM FORMULATION

To describe more specifically the application of the CE method to the problem under study, let us consider first a simpler problem, where demand is not random and all demands and penalties are identical. It is clear that in this case only the transportation costs matter; therefore, the problem reduces to the well-known travelling salesman problem (TSP). Notice that such simplification is done just to motivate the developments below the actual problems we consider become far more complicated than the TSP due to the introduction of random demands. The use of the CE method for the TSP has been widely studied in CE literature, see for instance de Boer et al. (2005) and Rubinstein (1999). Notice also that, when dealing with the TSP, one does not assign directly a probability to each possible route, since this would imply having n! variables; rather, one assigns transition probabilities to an appropriately defined Markov chain. This was realized by Rubinstein (1999), We describe now the algorithm in more detail.

We describe now the algorithm in more detail:
1. Let P be an initial transition probability matrix; set k := 1, ρ := 0.01.
2. Generate N valid routes $r^1, \ldots, r^N$ according to the transition probabilities in P, and compute the cost F of each route.
3. Let γk be the sample (1 − ρ)-quartile of the sequence $F(r^1), \ldots, F(r^N)$.
4. (*) If necessary, decrease ρ and/or increase N and go back to Step 2.
5. Update the transition probabilities as follows:

$$P_{uv} := \frac{\sum_{j=1}^{N} I\{(u.v) \in r^j\}.I\{F(r^j) \le \gamma k\}}{\sum_{j=1}^{N} I\{F(r^j) \le \gamma k\}} \tag{1}$$

6. Test stopping criterion; if satisfied, STOP; otherwise, let $k := k + 1$ and go back to step 2.

Step 4 of the above algorithm is in a sense optional, since it constitutes an enhancement to the algorithm. In fact, many of the reported implementations omit that step. In the context of rare event simulation, those updates are discussed in Homem-de-Mello and Rubinstein (2002).

5. BRANCH AND BOUND ALGORITHM

The branch and bound (BB) algorithm is the most widely used method for solving both pure and mixed integer programming (MIP) problem in practice. A pure integer program (PIP) is one were all the variables are restricted to be integer. A mixed integer program restricts some of the variable to be integer whereas others can assume continuous (fractional) value. Most commercial computer codes for solving integer programs are based on this approach. Basically the BB algorithm is just an eficient enumeration procedure for examining all posible integer feasible solution. The reason for considering integer programs is that many practical problems require integer solution (Philips, 1987).

Consider a MIP problem of the following form :

Maximize $Z = cx$ (2)

Subject to : $Ax = b$ (3)

$x \ge 0$ (4)

x_j is an integer for $j \epsilon I$,

where I is the set of all integer variables.

The algorithm for BB method may be summarized follow:

1. Solve the linear program by the simplex method.
2. Evaluate the optimum solution. If the base variable is the expected round means the optimum solution has been reached, thus the problem is solved.
3. a. If the base is still worth fractional variable, the variable will be branched into sub-problems.
   b. If there are several variable fractional-shaped base, then select the variable that has the largest fractional value.
   c. If there are several variables largest bases have the same fractional value, then select the variables arbitrarily.
4. Thorough optimum solution, if the base variable is the expected round means the optimum solution has been reached, thus the problem is solved.
5. For each sub-problem, the value of the optimum solution is obtained regardless of the integer limit set as the upper limit. Round best solution to the lower limit, which is the first solution to the LP rounded down to the maximization problem, and rounded up to the minimization problem. Sub-sub-problems which have an upper limit that is less than the lower limit exists, is not included in the subsequent analysis. A feasible solution is the same round good or better than the upper limit for each sub-problem is sought. If such a solution occurs, a sub-problem with the upper limit of the best selected for branched, and return to step 3.

The key aspects of BB algorithm may be summarized follow (Philips, 1987):
1. If it is unnecessary to branch on a subproblem, it is fathomed. The following three situations result in a subproblem beeing fathomed: (a). the subproblem is infeasible; (b). The subproblem yields an optimal solution in which all variables have integer values; and (c). The optimal Z value for the subproblem does not exceed the current lowe bound.
2. A Subproblem may be eliminated from consideration in the following situations : (a) the subproblem is infeasible; (b) the lower bound is at least a large as the Z value for the subproblem.

For BB Algorithm, TSP problem can be formulated as :

$$Min\ \ Z = \sum_i \sum_j c_{ij} x_{ij} \tag{5}$$

s.t.

$$\sum_{i=1}^{N} x_{ij} = 1\,(\,for\ \ j = 1,2,......,N) \tag{6}$$

$$\sum_{j=1}^{N} x_{ij} = 1\,(\,for\ \ j = 1,2,......,N) \tag{7}$$

$$u_i - u_j + Nx_{ij} \le N - 1\ \ (\,for\ \ i \ne j;\ \ i = 2,3,.......N;\ j = 2,3,.....N) \tag{8}$$

all x_{ij} = 0 or 1, all $u_j \geq 0$

6. TWO GROUPS 6 NODE CASE STUDY

Consider we have two groups TSP problem (Figure 1). A warehouse have two vehicles must schedule to visit 5 destinations, respectively. Each vehicle start and finish on the warehouse for loading. The initial solution is generated through a clustering scheme. Distance from one destination to other destination for vehicle 1 (Group A) and vehicle 2 (Group B), respectively, are shown on Tabel 1 and Tabel 2. We must schedule the route of each vehicle so the total distance for each vehicle are minimized.

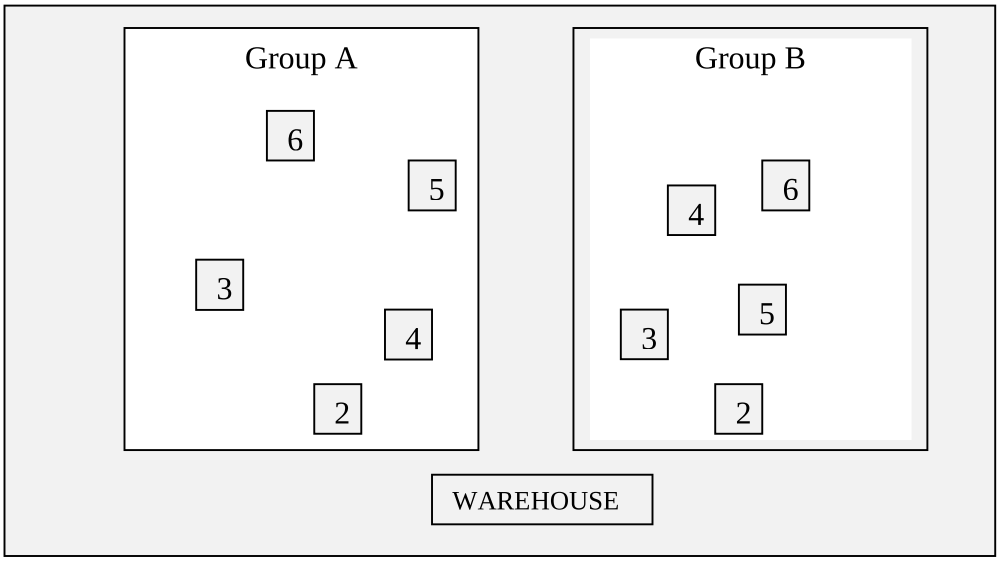


Figure 1. Group A and B TSP Problem

Tabel 1. Distance Node to Node (kilometers) Group A

Node	1	2	3	4	5	6
1	0	6	10	5	15	20
2	10	0	5	8	12	15
3	10	5	0	2	10	8
4	5	8	2	0	5	10
5	15	12	10	5	0	6
6	20	15	8	10	6	0

(source: measuring from city map)

Tabel 2. Distance Node to Node (kilometers) Group B

Node	1	2	3	4	5	6
1	0	3	5	8	10	6
2	3	0	10	5	5	8
3	5	10	0	6	9	10
4	8	5	6	0	6	6
5	10	5	9	6	0	8
6	6	8	10	6	8	0

(source: measuring from city map)

7. RESULT

7. 1. CE METHOD SOLUTION

We can solve the problem by CE Method with develop short program on Mathlab. Optimal route for Group A is **-2-5-6-4-3-1**, minimum total distance is 33. Optimal route for Group B is **1-4-5-6-3-2-1**, minimum total distance is 35. There are no alternate solution, i.e. no alternate route, for Group A and Group B. As brief before, the CE Method used iterative procedure to obtained optimal solution. Optimal solution was reached on the 4-iteration for Group A (Figure 2) and the value was remain to the next iteration. Hence, optimal solution for Group B was reached on the 4-iteration, and the next iteration was remain (Figure 3).

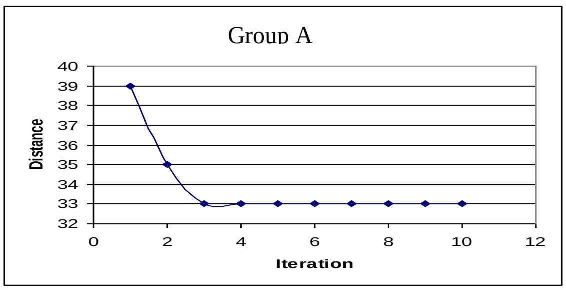


Figure 2. Iteration CE Method For Group A

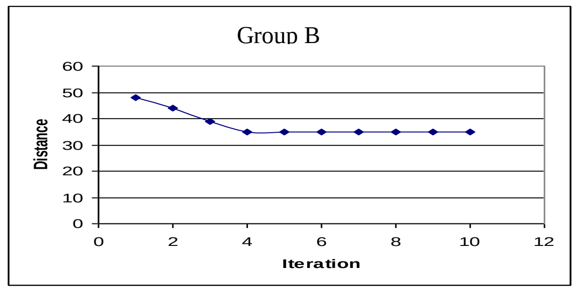


Figure 3. Iteration CE Method For Group B

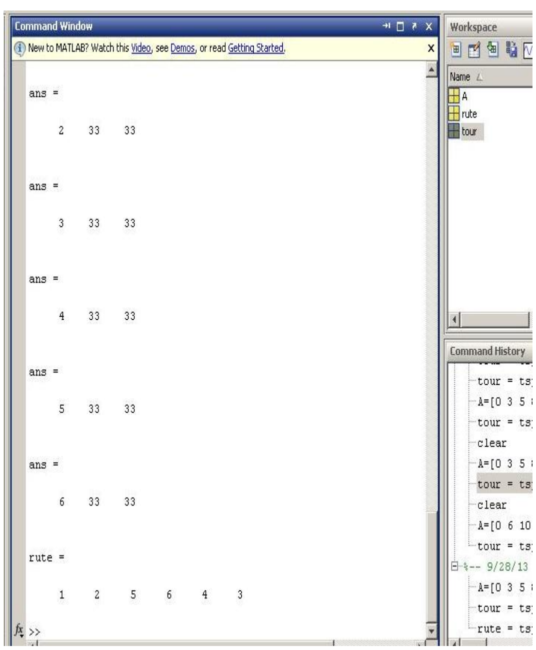


Figure 4. Output Matlab Program from from Group A

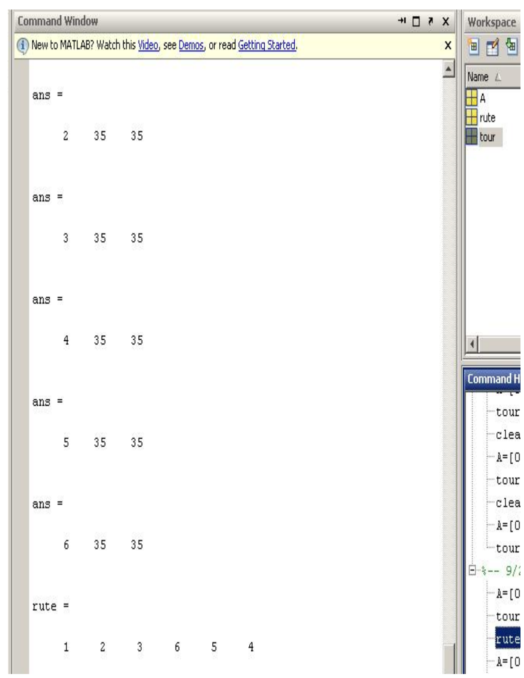
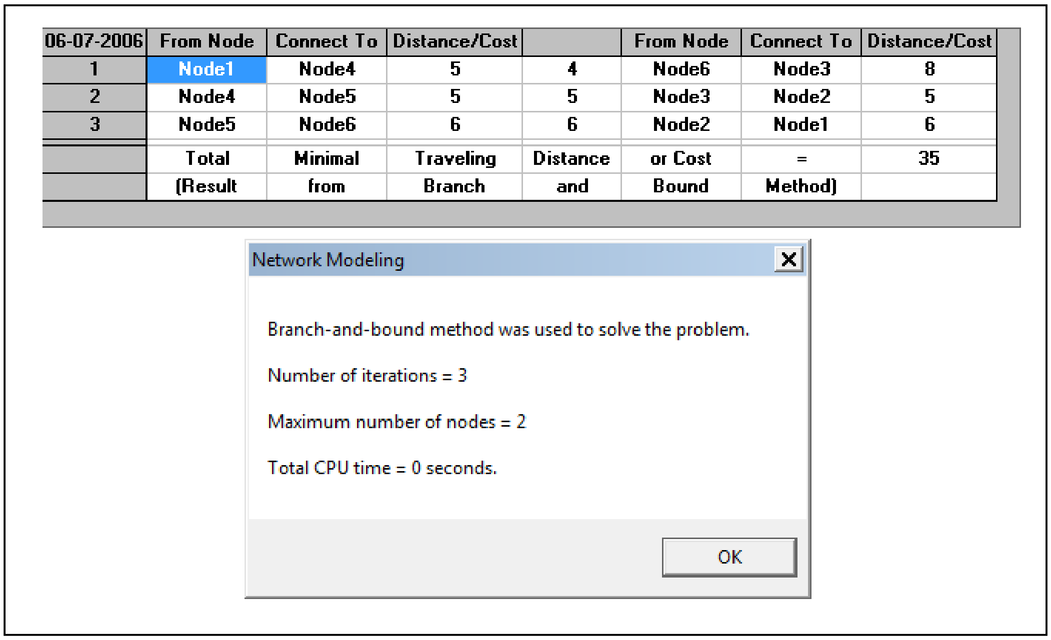


Figure 5. Output Matlab Program from Group B

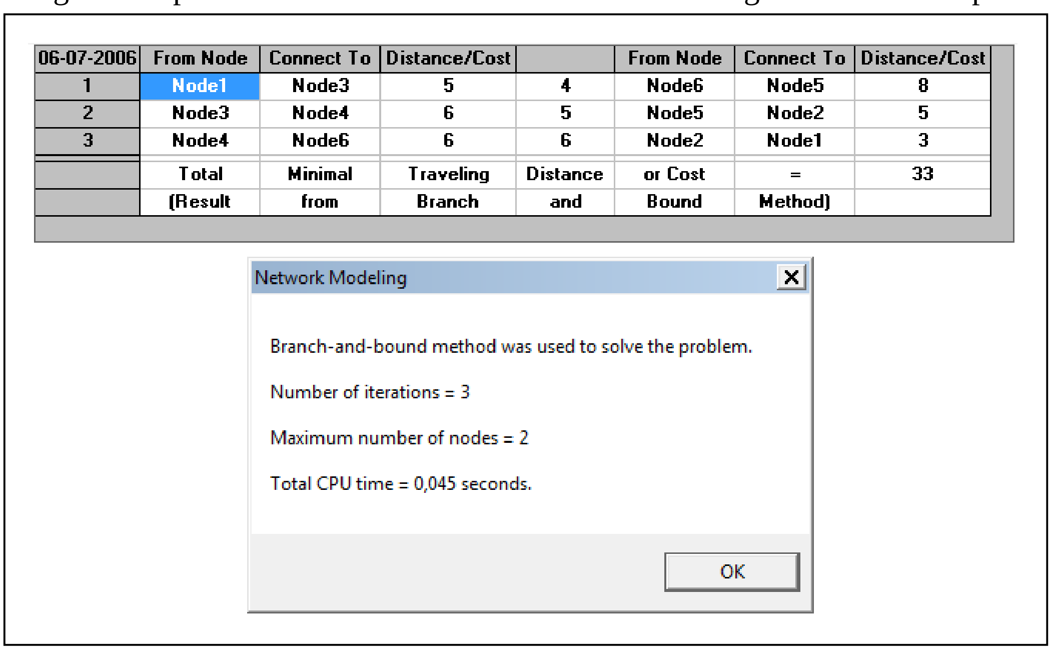
7. 2. BB ALGORITHM SOLUTION

We can solve the problem by BB Method with helped by Win QSB. Optimal route for Group A is **1-4-5-6-3-2-1**, minimum total distance is 35. Optimal route for Group B is **1-2-5-6-4-3-1**, minimum total distance is 33. There are no alternate solution, i.e. no alternate route, for Group A and Group B. BB Method used iterative procedure to obtained optimal solution. Optimal solution was reached on the 3-iteration for Group A (Figure 4). Hence, optimal solution for Group B was reached on the 3-iteration, and the next iteration was remain (Figure 5).



06-07-2006	From Node	Connect To	Distance/Cost		From Node	Connect To	Distance/Cost
1	Node1	Node4	5	4	Node6	Node3	8
2	Node4	Node5	5	5	Node3	Node2	5
3	Node5	Node6	6	6	Node2	Node1	6
	Total	Minimal	Traveling	Distance	or Cost	=	35
	(Result	from	Branch	and	Bound	Method)	

Figure 6. Optimal Solution and Number Iterative BB Algorithm For Group A



06-07-2006	From Node	Connect To	Distance/Cost		From Node	Connect To	Distance/Cost
1	Node1	Node3	5	4	Node6	Node5	8
2	Node3	Node4	6	5	Node5	Node2	5
3	Node4	Node6	6	6	Node2	Node1	3
	Total	Minimal	Traveling	Distance	or Cost	=	33
	(Result	from	Branch	and	Bound	Method)	

Figure 7. Optimal Solution and Number Iterative BB Algorithm For Group B

7. 3. COMPARISON OPTIMAL SOLUTION CE METHOD AND BB ALGORITHM

CE Method and BB Algorithm can obtain the same optimal route for vehicle 1 and vehicle 2, as display on Figure 6. We see that two method use two differens procedure/algorithm, but can obtain the same solution. Two method use iterative procedure to obtain optimal solution. CE method has 4 iteration for Group A dan B, and BB algorithm need 3 iteration.

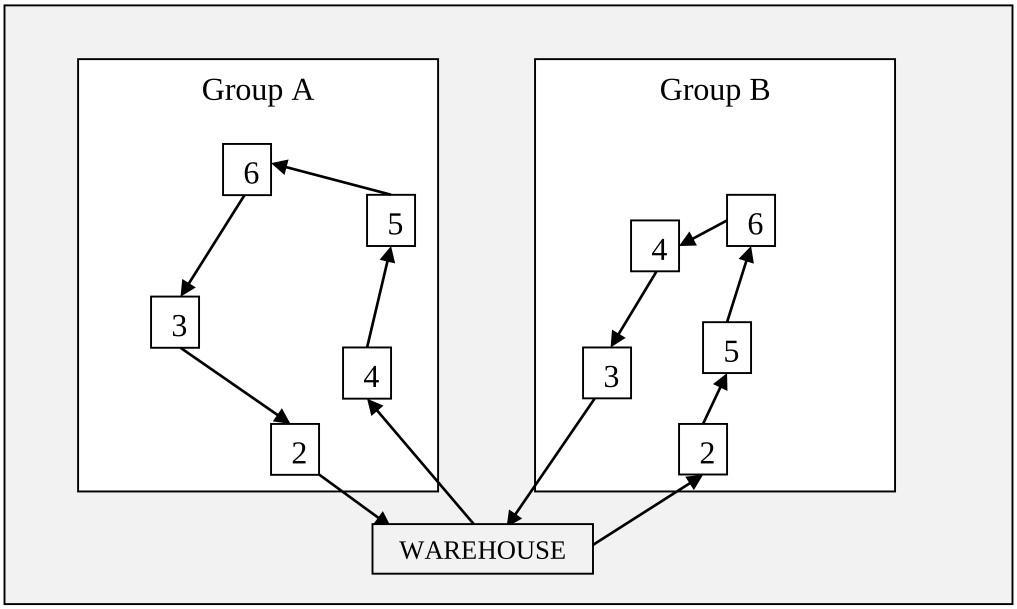


Figure 8. Optimal Route for Group A and Group B with CE Method and BB Algorithm

8. CONCLUSION

Vehicle flow routing problem was transformed into a combinatorial optimization problem in this paper. The elements of the problem are all optional feasible flow lines. The optimal solution stands for the streamline organization scheme. The Cross-Entropy as the metaheuristic approach is employed in this paper, and compared with the Branch and Bound Algorithm, as the analytical approach. The results indicate that is simple and easy to realize both methods. The two method can obtain similar solution with number iteration similar relatively. For CE, the another advantage of the method is that the results are stable in repeat calculation.

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